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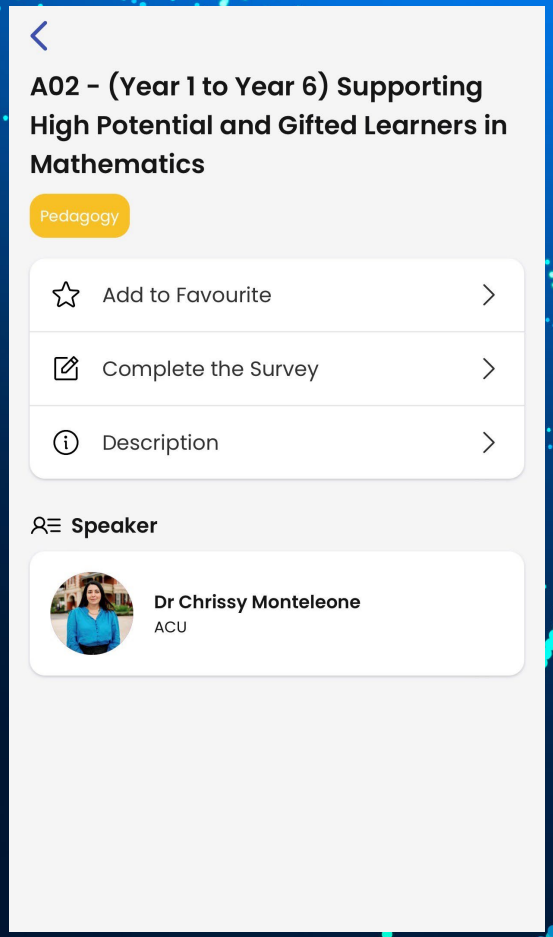

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How to help students visualise transformations using technology

Presenters

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Acknowledgement to the country

I would like to acknowledge the traditional owners of the land on which we meet today and pay my respects to their Elders past and present and extend that respect to other Aboriginal and Torres Strait Islander people who are present today.

Our aim

In this workshop we will present the strategies we use to help our students understand how transformations work. These strategies can be used with year 10 and year 11 students studying Mathematics Methods. We will be making connections, through various examples, between function notation and mapping equations using visual aids (DESMOS and GeoGebra). Multiple ways will be identified to obtain the same image function. Some general cases will be covered through animations.

The plan for today

- Warm up activity
- Learning transformations step-by-step
- Strategies and activities
- Resources

Warm-up activity

The paper strips show steps in learning transformations.

Order them according to the sequence in which learning transformations should occur.

There are blank ones for you to add other steps if necessary.

Warm-up activity

Is any of these steps unnecessary?

Are there any steps / aspects missing?

Have you been surprised by any step listed?

Learning transformations step-by-step

- Defining transformations
- Transformation using coordinates of a point **(mapping-Worksheet)**
- Transformations using rules and function notation
- Order matters
- Describing transformations from rules
- Describing transformations using mapping
- Applying transformations and miscellaneous questions

Transformations using rules and function notation

- Example 1: Transformation Part 1 | Desmos
- Example 2: Transformations - Basics • Activity Builder by Desmos Classroom
- Extension: Which parabola? | Quadratics | Underground Mathematics

Order matters

Original: $f(x) = x^2$

Find the equation of the image after the following sequence of transformations. In each case write down the mapping equations.

Option 1

1. Translation 3 units to the right
2. Dilation by a factor of 2 from the y axis

Option 2

1. Dilation by a factor of 2 from the y axis
2. Translation 3 units to the right

Order matters 1 | Desmos

Is it important to show students different ways to describe transformations?

Describing transformations from rules

Task 1

Original: $f(x) = x^2$

Image: $g(x) = (2x - 5)^2$

Find four different sequences of transformations that map $f(x)$ onto $g(x)$.

Describing transformations from rules

Original: $f(x) = x^2 \therefore y = x^2$

Image: $g(x) = (2x - 5)^2 \therefore y' = (2x' - 5)^2$

Option 1&2

$$2x' - 5 = x \;\&\; y' = y \therefore x' = \frac{1}{2}x + \frac{5}{2} = \frac{1}{2}(x + 5) \& y' = y$$

1. Dilation by a factor of $\frac{1}{2}$ from the y axis followed by translation by $\frac{5}{2}$ units to the right $g(x) = \left(2\left(x - \frac{5}{2}\right)\right)^2$
2. Translation 5 units to the right followed by dilation by a factor of $\frac{1}{2}$ from the y axis $g(x) = (2x - 5)^2$

Describing transformations from rules

Original: $f(x) = x^2 \therefore y = x^2$

Image: $g(x) = (2x - 5)^2 \therefore y' = (2x' - 5)^2 = 4\left(x' - \frac{5}{2}\right)^2$

$$\therefore \frac{y'}{4} = \left(x' - \frac{5}{2}\right)^2$$

Option 3 &4

$$x' - \frac{5}{2} = x \;\&\; y' = 4y \therefore x' = x + \frac{5}{2} \;\&\; y' = 4y$$

1. Dilation by a factor of 4 from the x axis followed by translation by $\frac{5}{2}$ units to the right
2. Translation by $\frac{5}{2}$ units to the right followed by dilation by a factor of 4 from the x axis

Describing transformations using mapping

Example 1:

The function $y = f(x) = x^3$ is mapped onto $y = g(x)$ following the sequence of transformations given below:

1. Translation 3 units up
2. Dilation by a factor of 2 from the x axis
3. Translation 4 units to the right
4. Reflection in the y axis
5. Dilation by a factor of 3 from the y axis

Find the rule of $g(x)$.

Describing transformations using mapping

Solution (method 1- based on rules)

$f(x) = x^3$

$\rightarrow h(x) = f(x) + 3 = x^3 + 3$

$\rightarrow k(x) = 2h(x) = 2(x^2 + 3) = 2x^3 + 6$

$\rightarrow l(x) = k(x - 4) = 2(x - 4)^3 + 6$

$\rightarrow m(x) = l(-x) = 2(-x - 4)^3 + 6$

$\rightarrow n(x) = m\left(\frac{1}{3}x\right) = 2\left(-\frac{1}{3}x - 4\right)^3 + 6 = g(x)$

Transformations Example 1 | Desmos

Describing transformations using mapping

Solution (method 2- based on mapping)

1. Translation 3 units up $y' = y + 3$
2. Dilation by a factor of 2 from the x axis $y' = 2(y + 3)$
3. Translation 4 units to the right $x' = x + 4$
4. Reflection in the y axis $x' = -(x + 4)$
5. Dilation by a factor of 3 from the y axis $x' = -3(x + 4)$

Find the rule of $g(x)$.

Describing transformations using mapping

Solution (method 2 continues)

$$x' = -3(x+4) \therefore x = -\frac{1}{3}(x'-4)$$

$$y' = 2(y+3) \therefore y = \frac{1}{2}(y'-6)$$

$$y = x^3 \text{ becomes } \frac{1}{2}(y'-6) = (-\frac{1}{3}(x'-4))^3$$

same as

$$y' = 2\left(\left(-\frac{1}{3}(x'-4)\right)^3 + 3\right)$$

$$-2\left(\frac{1}{3}x+4\right)^3 + 6 = g(x)$$

Describing transformations using mapping (if time)

Task

The domain of function $y = f(x)$ is $[m, n]$ and its range is $[u, v]$ where $m, n, u, v \in \mathbb{R}$.

State the domain and range of the function:

$g(x) = 3f(-2x + 4) - 2.$

Describing transformations using mapping

Solution

The transformations applied to $y = f(x)$ to be mapped onto $y = g(x)$ are as follows:

For x:

1. Translation by 4 units to the left
2. Dilation by a factor of $\frac{1}{2}$ from the y axis
3. Reflection in the y axis.

Same as: $x' = -\frac{1}{2}(x - 4)$

Describing transformations using mapping

Solution continues

For y:

1. Dilation by a factor of 3 from the x axis
2. Translation 2 units down.

Same as: $y' = 3y + 2$

Describing transformations using mapping

Solution continues

Same transformations apply to the domain and range of $f(x)$.

Domain of $g(x)$ is: $[-\frac{1}{2}(m-4), -\frac{1}{2}(n-4)]$

Range of $g(x)$ is: $[3u+2, 3v+2]$

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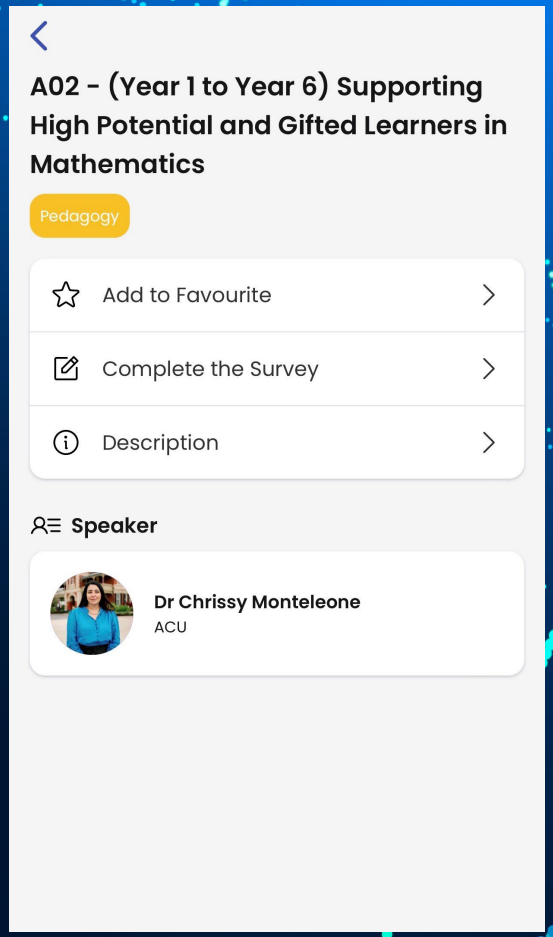

Defining transformations 1

Which quadratic? | Quadratics | Underground Mathematics

Defining transformations 2

Transformers | Combining Functions | Underground Mathematics

Transformations using coordinates of a point

- Blue Point Rule • Activity Builder by Desmos Classroom